

Fundamental Concepts in Mathematical Programming



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ARKANSAS

Laxmi Prasad Poudel



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Origin of Mathematical Programming

- “A mathematical method to **allocate scarce resources** to competing activities in an **optimal manner** when the problem can be expressed using a **linear objective function** and **linear inequality constraints**”
- Mathematical Programming entails:
 - **Formulation** of real-world problems in mathematical terms
 - Development of **techniques** for solving the models
 - Involves **optimization** and should capture **key features** of a complex **decision problem**
 - Popular mathematical programming approaches: **Linear programming (LP)** and **mixed-integer linear programming (MILP)**
- Linear programming became widely popular after the World War II in 1947



Fundamentals of Linear Programming

- LP is an extremely **general** technique limited mainly by imagination
- **Decision Variable** → Set of quantities that need to be determined
- **Objective Function** → Indicate how each variable contributed to the value to be optimized → Maximize or minimize some linear equation

$$\text{maximize or minimize } Z = \sum_{i=1}^n c_i X_i$$

where c_i = the objective function coefficient corresponding to the i^{th} variable, and
 X_i = the i^{th} decision variable.¹

- **Constraints** → Define the possible values that variables may take (e.g., resource constraints, non-negative constraints)

$$\text{subject to } \sum_{i=1}^n a_{j,i} X_i \leq b_j \quad j = 1, 2, \dots, m$$

where X_i = the i^{th} decision variable,
 $a_{j,i}$ = the coefficient on X_i in constraint j , and
 b_j = the right-hand-side coefficient on constraint j .



Linear Programming- Problem Formulation

- Most difficult aspect of LP: Formulation of the problem
- Components of LP
 - What? → Decision Variables → Unknowns/Solutions
 - Why? → Objective Function → Motivation/Goal
 - Limits/Restrictions → Constraints → Resources/Demand
 - Data → Parameters/Sets → Money/amount of resources
- Steps:
 - Identify the decision variables;
 - Formulate the objective function; and
 - Identify and formulate the constraints
 - Adding non-negativity constraints



Linear Programming - Example

In his spare time, my younger brother has picked up the hobby of making homemade bread like in the picture below.



To earn some extra money, he decides to start selling two of his favorite kinds of bread: The Saturday White Bread and Baguettes. Both of these breads only have four ingredients: flour, water, salt, and yeast.

For two loaves of Saturday White bread, he needs 1000g of flour, 720g of water, 21g salt, and 4g yeast. For five baguettes, he needs 1000g of flour, 800g of water, 15.8g of salt, and 5.9g of yeast.

He recently visited his local Sam's Club and has in his kitchen 9060g of flour, unlimited water, 1020g of salt, and 454g of yeast.

His currently sells The Saturday White Bread for \$5/loaf so a batch of 2 sells for \$10. He sells each baguette for \$3 each, thus, a batch of 5 sells for \$15. He just started selling baguettes and only has demand for 10 total. How many batches of each bread should he make such that he maximizes his revenue?

- Decision variables

- $x_1 \rightarrow$ # of batches Saturday White Bread
- $x_2 \rightarrow$ # of batches of Baguettes

- Objective Function

$$\text{maximize } 10x_1 + 15x_2$$

- Constraints

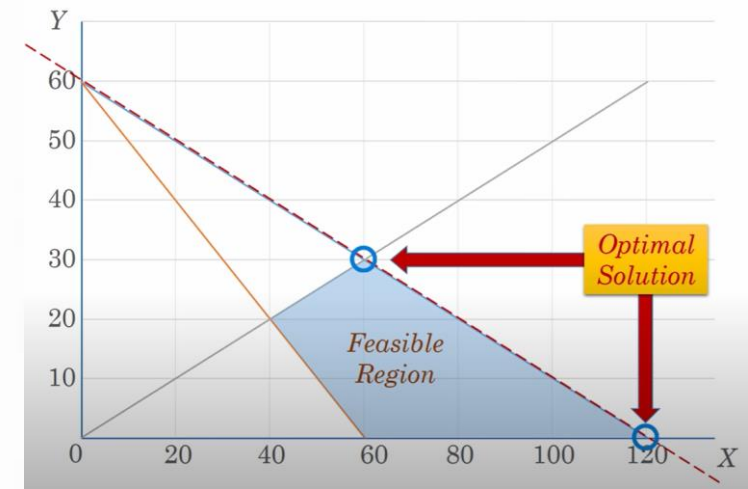
- Flour: $1000x_1 + 1000x_2 \leq 9060$
- Salt: $21x_1 + 15.8x_2 \leq 1020$
- Yeast: $4x_1 + 5.9x_2 \leq 454$
- Baguettes: $x_2 \leq 2$
- Non-negativity: $x_1, x_2 \geq 0$

Credit: Dr. Sarah Nurre @UofA



Linear Programming - Solution Approach

- LPs can be solved using different approaches
- Graphical Method
- Simplex Method
- Large scale-mathematical solvers
 - Cplex
 - Gurobi
 - FortMP
 - NMath



Simplex Algorithm

1. Formulate the Problem
2. Find out the Initial Solution
3. Test for Optimality
4. Test for Feasibility
5. Identify the Pivot Element (Key Element)
6. Determine the New Solution
7. Revise the Solution



Linear Programming - Demo for multi-robot scheduling

Demo using Python with Cplex using Jupyter Notebook

